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Original Paper

A Study on Real-time Predictions of Flood Runoff Using Spatially Distributed Rainfall Data

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Abstract

A new method for real-time predictions of flood runoff using effective information of rainfall spatially distributed was developed. In this method, a single basin of moderate size was divided into sub-basins so that the rainfall characteristics could be treated as uniform in each sub-basin. The method consists of two stages. One is to update the state vectors in the sub-basins by using the filtering theory, the other is to compose runoff from the sub-basins as the total flood runoff by using the kinematic wave theory. An actual river basin was used for verification of the method. Results of the application showed that this method had better capability than the simple method of thinking of the basin as a single unit. The influences of rainfall forecasting and discharge measurement accuracy on flood prediction were also discussed with simulation data generated by the method.

1. Introduction

The filtering theory can provide a powerful means for real-time prediction of flood runoff. Many investigations have dealt with developing a more effective algorithm for prediction by using the filtering theory. There are several possible ways to set up an appropriate hydrologic model and to select the state vector in applying the theory to a runoff system. Hino (1972) has showed an application of the linear filtering theory with the linear response model to runoff prediction. Nishimura et al. (1977), and Hoshi and Yamaoka (1980) have used the non-linear filtering theory with the storage model. Kitanidis and Bras (1980, a, b, c) have discussed the real-time prediction of including the transient errors with the conceptual model. Most of the investigations so far have been concerned with cases of spatially uniform rainfall in a basin.

Recently, in Japan, the network of radar raingauge is getting to be able to cover the whole land area of the country. The network can bring about real-time data of spatially distributed rainfall intensity. It has therefore become imperative to study advanced methods which will enable more accurate flood prediction using the newly available rainfall measurement data.

This study aims at the following two points. One is to develop a new method for real-time flood prediction by using the spatially distributed rainfall data. The other is to discuss the influence of rainfall forecasting and discharge measurement accuracy on flood prediction. In this paper, a whole runoff system composed of sub-systems corresponding to the characteristics of the distributed rainfall is examined. Then the total runoff of the system is integrated by taking into account the concentration time from each sub-system to the end of the whole system.

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Many rainfall-runoff models have been proposed. However, some basic and simple models such as the unit graph method have been used for flood prediction and warning in practical emergence operation. As the first step towards developing a new complete method, therefore, the linear response function of each sub-system is taken as the state vector to be estimated in this paper.

2. Runoff system composed of sub-systems

(a) Division of a river basin

In most cases, the catchment area of a river basin for which we need to give a flood prediction is larger than the characteristic scale of the spatial variation of rainfall. For making better use of the data by the radar raingauge measurement, for example if spatial resolution is 3 km, it may be more effective to divide the basin into reasonably small basins corresponding to the characteristic scale.

If we can get a set of observed discharge data at each sub-basin, the sequential filtering algorithm proposed by Tamura and Ueno (1971) can be used for the total flood runoff prediction. However, it is difficult in general to get such data at the intermediate points of the basin. This study deals with the case in which runoff discharge is observed only at the end of the whole basin. At the same time, however, the number of state vectors to be estimated increases. Thus the case of prediction by treating the whole as a composed basin with the distributed rainfall data and the other case of prediction by treating it as a single basin with average rainfall data should be compared.

(b) Linear sub-system and composition

Assume a linear time-varying system given by

$$\mathbf{x}(k+1)=\mathbf{F}(k)\mathbf{x}(k)+\mathbf{G}(k)\mathbf{u}(k) \qquad (1)$$

$$z(k)=\mathbf{M}(k)\mathbf{x}(k)+v(k) \qquad (2)$$

where $\mathbf{x}$ is the state vector of the system, $\mathbf{F}(k)$, $\mathbf{M}(k)$ and $\mathbf{G}(k)$ are known system matrices, $z(k)$ is scalar measurement, $\mathbf{u}(k)$ is zero mean vector white noise process independent of $\mathbf{x}(k)$ and with known covariance matrix $\mathbf{U}(k)$, and $v(k)$ is a zero mean white noise process independent of $\mathbf{x}(k)$ and with variance $V(k)$.

If $\mathbf{x}(k)$ is assumed as the discrete linear response function as used by Hino (1971), equation (1) is simplified as follows:

$$\mathbf{x}(k+1)=\mathbf{I}(k)\mathbf{x}(k)+\mathbf{u}(k) \qquad (3)$$

where $\mathbf{I}(k)$ is the unit matrix.

In such a situation the Kalman filter formulation gives

$$\hat{\mathbf{x}}(k \mid k)=\hat{\mathbf{x}}(k \mid k-1)+\mathbf{K}(k)e(k) \qquad (4)$$

where $\hat{\mathbf{x}}(k+1 \mid k)$ represents the estimate of the state of the system at time $k+1$, given observations up to time $\mathbf{k}$, $K(k)$ is a Kalman gain matrix to be defined later, and $e(k)$ is the step k prediction residual:

$$e(k)=z(k)-\mathbf{M}(k)\hat{\mathbf{x}}(k \mid k\text{-}1) \qquad (5)$$

The gain matrix $\mathbf{K}(k)$ results from minimizing the mean square error of estimation and takes the form

$$\mathbf{K}(k+1)=\mathbf{P}(k+1 \mid k+1)\mathbf{M}^{T}(k+1)V^{-1}(k+1) \qquad (6)$$

where $\mathbf{P}(k+1 \mid k+1)$ is the mean square error of estimation matrix at time $k+1$, given observation to time $k+1$:

$$\mathbf{P}(k+1 \mid k+1)=E\left[(\mathbf{x}(k+1)-\hat{\mathbf{x}}(k+1 \mid k+1))(\mathbf{x}(k+1)-\hat{\mathbf{x}}(k+1 \mid k+1))^{T}\right] \qquad (7)$$

The mean square error of estimation obeys the conventional recursive relationship (Jazwinski (1968)).

The term $\mathbf{M}(k)\mathbf{x}(k)$ in equation (2) for the two cases of single basin and composed basin is expressed as follows:

<single basin> :

$$\mathbf{M}(k)\mathbf{x}(k)=[r(k),\ r(k-1),\ ...\ r(k-t_m)]\,[x(k),\ x(k-1),\ ...\ x(k-t_m)]^{T}$$

(8)

where r(k) is rainfall intensity averaged over the whole basin at time k, t_m is maximum response time to be considered in the basin.

<composed basin> :

$$\mathbf{M}(k)\mathbf{x}(k)=\mathbf{M}_1(k)\mathbf{x}_1(k)+\mathbf{M}_2(k)\mathbf{x}_2(k)+...+\mathbf{M}_i(k)\mathbf{x}_i(k)+...+\mathbf{M}_n(k)\mathbf{x}_n(k) \quad (9)$$

where $\mathbf{x}_i(k)$ is the state vector of i-th sub-basin at time k, $\mathbf{M}_i(k)$ is the time series of observed rainfall at the i-th sub-basin which is set accounting for the time lag from the sub-basin to the end of the whole basin.

The time lag of each sub-basin can be determined approximately by the kinematic wave theory along the main river channel. The end of each sub-basin is connected to the main river. The surface roughness, slope and flow regime of the river are assumed invariant in space and time.

The characteristics of flood propagation along the river are expressed as following equations.

$$dx/dt=ma^{1/m}\,Q^{1-1/m} \quad (10)$$

$$dQ/dx=q \quad (11)$$

constant relating to the resistance law of flow, m = 5/3 in Manning's formula, Q is the flow discharge of the river, q is the lateral inflow rate per unit length along the river, and a is given as follows :

$$a=I^{1/2}\,/\,(nB^{2/3}) \quad (12)$$

where I is the slope of the river bed, n is the Manning's roughness coefficient and B is the width of the river channel.

Let us apply the theory to the runoff system approximately with rough discrete expression. Referring to the notations illustrated in **Figure 1,** if we make the lateral inflow,ΔQ, correspond to the

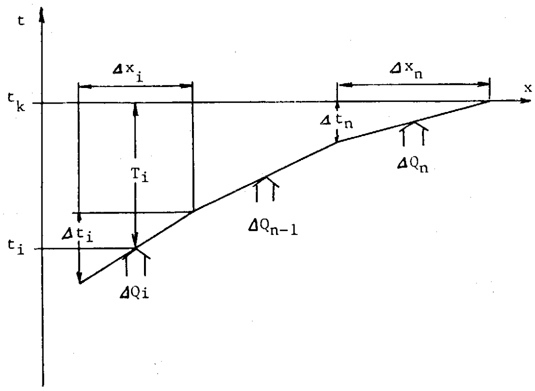


Fig. 1. Concentration time T and composition of runoff discharge

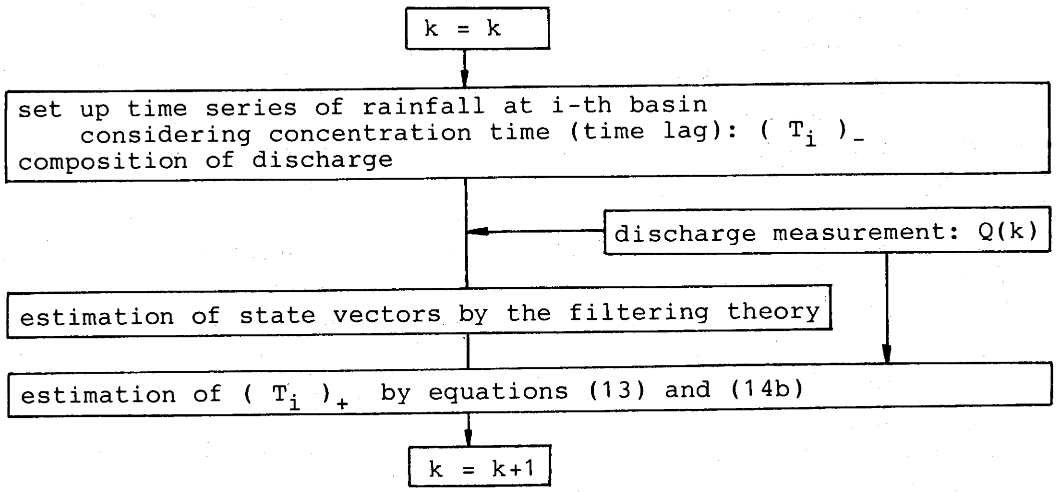


Fig. 2. Outline of the calculation procedure.

where x is distance along the river, t is time, m is a runoff discharge at the end of i-th sub-basin, q, the following successive expressions can be given.

$$\Delta t_i = \Delta x_i / (ma^{1/m} Q_i^{1-1/m}) \tag{13}$$

$$\Delta Q_i = q\Delta x_i \tag{14a}$$

$$\Delta Q_i = Q_{i+1} - \Delta Q_{i+1} \tag{14b}$$

If the total discharge at the end of the whole basin, Q_n is estimated at the time step k, the concentration time T_i of each sub-basin is estimated by equations (13) and (14b) successively using the updated value of ΔQ_i. In the calculating practice, the time lags are determined after the estimation of the state vectors by the filtering theory. And then, time series of rainfall intensity is set up for the next time k+1 considering the time lags of sub-basins. **Figure 2** illustrates the outline of the calculation procedure.

3. Application of the method to the experimental basin

(a) Division of the Kanna River basin

The new method developed above was applied to flood prediction at the Kanna River experimental basin which has a catchment area 374 km². Rainfall data have not been obtained by a radar raingauge. However, high quality rainfall and runoff data have been acquired for hydrological research. The basin is divided into six sub-basins considering characteristics of topography and rainfall. **Figure 3** illustrates the Kanna River basin and its sub-basins. Time series of rainfall intensity at each sub-basin are shown in **Figure 4.**

In the application, the following three cases of fundamental situation of prediction are examined.

Case (1) : single basin using equation (8).

Case (2) : composition of sub-basins using equation (9) with constant time lags; The constant time lags were given by the Rziha's empirical formula.

Case (3) : composition of sub-basins using equation (9) combined with equations (13) and (14b); the developed method described above; Referring to the topographic map, the parameters in equation (10) were assumed as; n=0.05, m=5/3, B=60m, and I = 0.006~0.01.

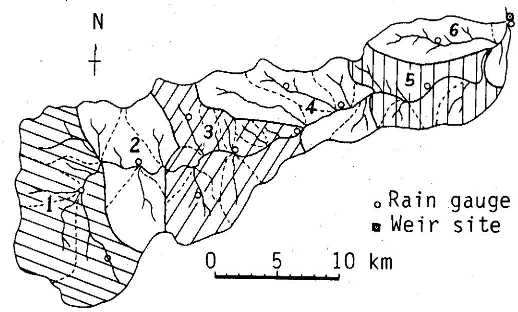


Fig. 3. The Kanna River experimental basin and its sub-basins.

In all cases, variance U_k was assumed 0.0001 as a standard value.

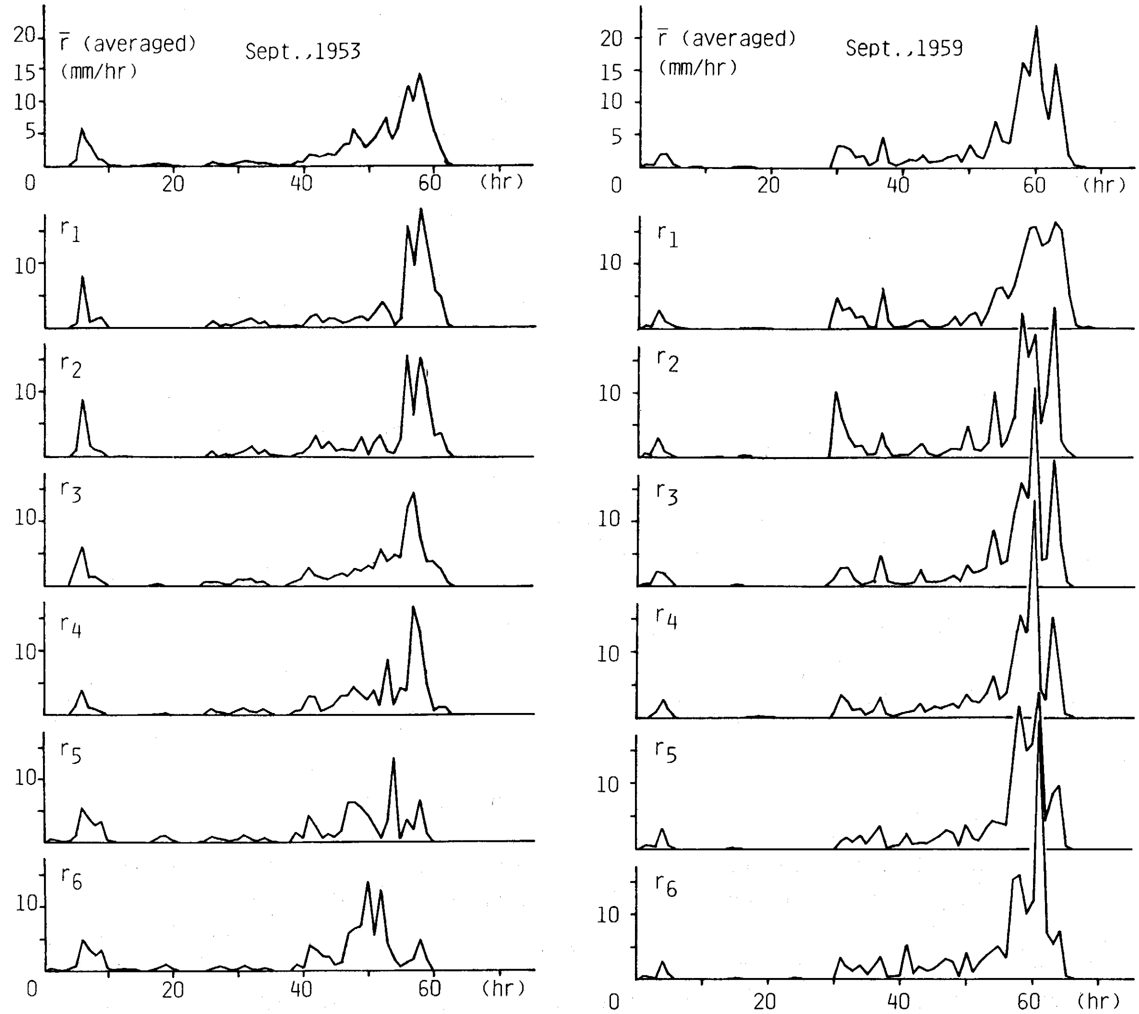
(b) Results of the application

Figure 5(a) shows the results of the predictions of the flood event in September, 1953. It seems difficult to find any distinct difference among them. On the other hand, the prediction of another relatively large flood event in september, 1959 is illustrated in **Figure 5(b).** It is clear that the prediction by the case (3) shows better agreement with the observed discharge than the others.

Figures 6 (a) and **(b)** illustrate variances of the prediction errors for the flood events in 1953 and 1959 respectively. And **Figure 6 (c)** shows the variance in the case of U_k=0.0007 in the event in 1959.

Comparing the results represented in the Figures 5 and 6, it can be seen that the new method developed here, the case (3), is of great advantage for predicting larger flood discharge. In other word, the method can take into account the non-linear effect of composing sub-basins by considering the reasonable concentration times along the river.

Estimated state vector at sub-basin No. 1 is exemplified in **Figure 7.** Figures 7 (a) and (b) illustrate the linear response function estimated at time steps from 65 to 68, near the peak discharge, by the two cases. The true response function is unknown. However, the shape of the function estimated by the



(a) Case of the flood event in 1953.

(b) Case of the flood event in 1959.

Fig. 4. Time series of rainfall intensity at each sub-basin.

case (3) is more similar to the patterns which have been obtained in many other river basins.

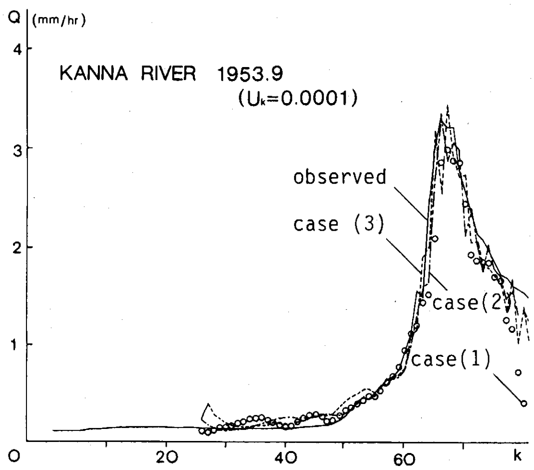
4. Influence of accuracy of both rainfall forecasting and discharge measurement on the flood predictions.

Many attempts have been made for developing rainfall forecasting using information of radar raingauge. Takasao et al. (1982) have dealt with the influence of accuracy of rainfall forecasting on flood prediction. Discharge measurement is one of the most important factors in flood predictions, but its accuracy is uncertain in most cases. It is, therefore, necessary to examine the influence of accuracy of both rainfall forecasting and discharge measurement on flood prediction at the same time.

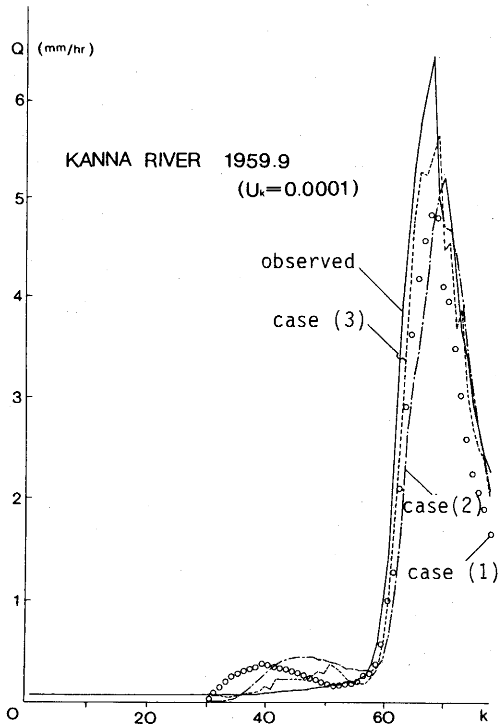
As a first step forwards the discussion of the influence mentioned above, it is useful to use a set of simulation data given true values.

(a) Data generation

Given that a 300 km^2 basin is composed of six sub-basins, runoff data for given rainfall can be generated inversely by the method developed above.



(a) Case of the flood event in 1953.



(b) Case of the flood event in 1959.

Fig. 5. Results of the flood predictions (lead time: 1hr).

The parameters of the response functions and the river channel were assumed as in **Table 1.**

(b) Presentation of rainfall forecasting

Based on dynamic, statistic and kinetic situations, several methods of short time rainfall forecasting have been proposed (Tatehira (1980)). In this study, the following simple and representative types of rainfall forecasting and accuracy of discharge measurement were examined.

Method (1): case of the forecasting of rainfall intensity; By referring to the true rainfall and using the index of forecasting accuracy proposed by Takasao et al. (1982), the forecasted rainfall is generated as,

$$E\ [\hat{r}_{k+l}] = r_{k+l},\quad Var\ [\hat{r}_{k+l}] = A_1^2 l\, r_{k+l}^2 \tag{15}$$

where r_{k+l} and $\hat{r}_{k+l}$ are the true and the forecasted rainfall at l step after time k respectively, E [] and Var[] are the operators of expectation and variance.

Method (2): case of the forecasting of average rainfall intensity through lead time;

$$E\ [\hat{r}_{m,k+l}] = r_{m,k+l},\quad Var\ [\hat{r}_{m,k+l}] = A_2^2 l\, r_{m,k+l}^2 \tag{16}$$

where $\hat{r}_{m,k+l}$ is the forecasted rainfall intensity averaged through lead time l.

By using equations (10), (11) and random numbers, the two types of forecasted rainfall can be simulated at each sub-basin. **Figure** 8 shows the two types of rainfall forecasting.

Accuracy of discharge measurement:

$$E\ [v_k] = 0.0,\quad V_k = Var\ [v_k] = S_Q^2\, Q^2 \tag{17}$$

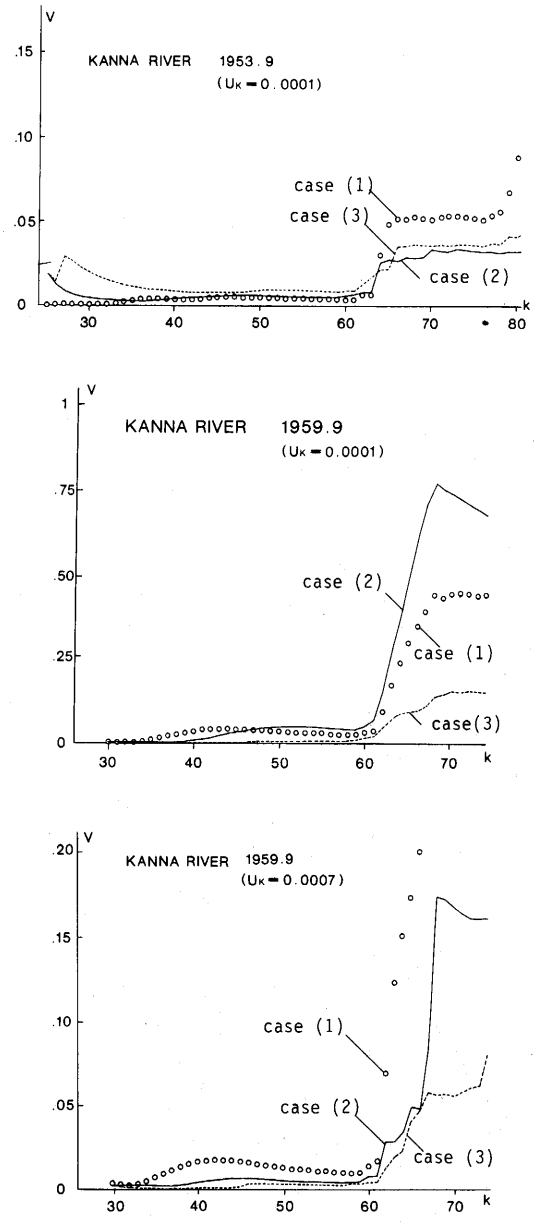


Fig. 6. Variances of the flood prediction errors (lead time: 1hr).

where Q is discharge at the end of the whole basin and S_Q is the parameter expressing the accuracy of discharge measurement.

(c) Results and discussions

Figure 9 illustrates an example of the flood predictions. The parameter A1 should not be compared directly with A_2. However, it can be recognized that the prediction by the method (1) shows distinctively better agreement with the observed, true, discharge than that by the method (2). This indicates that the forecasting of time series of rainfall intensity is needed for better flood prediction.

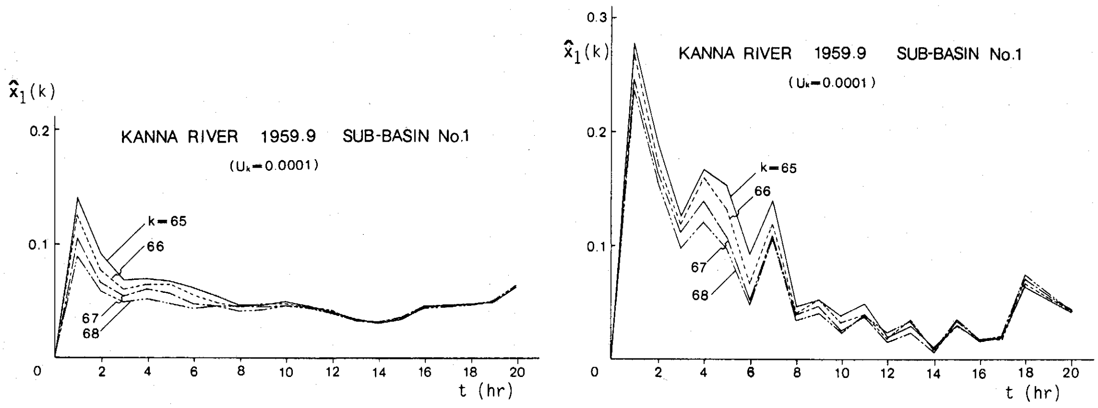
Changes in the accuracy of flood prediction for various lead times are shown in **Figures 10** to **13.** The variance of prediction error $\bar{\mathbf{W}}_{65}$ is taken as an index of the accuracy. $\bar{\mathbf{W}}_{65}$ means averaged value of the variances at the steps from k = 50 to 65, before and after the peak discharge. In **Figures 10** and **11,** when A_1 or A_2 takes a small value the influence depends entirely on the accuracy of discharge measurement S_Q. As S_Q increases more than about 0.2, $\bar{\mathbf{W}}_{65}$ increases distinctively in most cases. This fact suggests that the coefficient of variance of discharge measurement error should be kept within 0. 1, 10%, for flood prediction.

Figures 12 and **13** give inversely the influence of the accuracy of rainfall forecasting at the two conditions of discharge measurement. As S_Q increases, the effect of A_1 or A_2 on $\bar{\mathbf{W}}_{65}$ becomes less clear. This means that more accurate forecasting and measurement are required for better flood predictions. In the region of longer lead time, more than 4 hours, in the case by method (2), the prediction error increases rapidly. For this reason, it can be indicated that the total amount of forecasted rainfall in the region tends to be different from the true value in accordance with increasing A_2.

$\bar{\mathbf{W}}_{65}$ for various A_1 or A_2 are almost the same at the region of lead time within 5 hours in the method (1) or 3 hours in the method (2). This can be explained as follows : The method of prediction uses the distributed rainfall forecast with the time lag at each sub-basin, so the relative weight of the forecasting error at the upper sub-basins is reduced.

5. Concluding remarks

A new method for flood predictions taking into account spatially distributed rainfall data was developed. The method has the reasonable situation based on both stochastic aspects and physical mean-



(a) Case of constant concentration time. (b) Case of the kinematic wave theory.

Fig. 7. Linear response functions estimated at sub-basin No. 1.

Table 1. Parameters for the data generation.

sub-basin No. i	area (km^2)	respose func. parameter c_i	channel			
			length (km)	slope	width (m)	roughness n
1	50	1.2	7.0	0.01	30	0.05
2	50	1.0	7.0	0.009	40	0.05
3	50	0.8	7.0	0.009	50	0.05
4	50	0.6	7.0	0.008	60	0.05
5	50	0.5	7.0	0.008	70	0.05
6	50	0.5	7.0	0.008	80	0.05

response function: $h_i=0.5c_i^3t^2\exp(-c_it)$

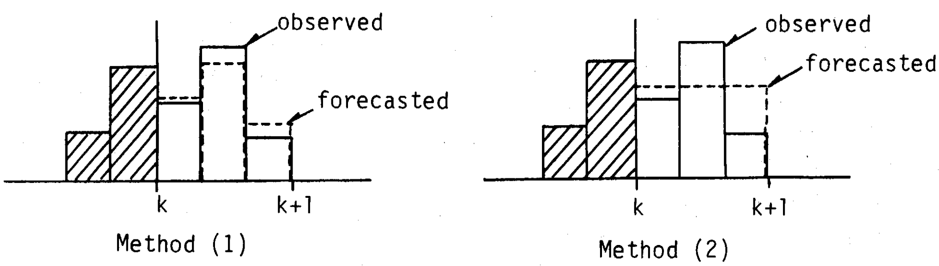
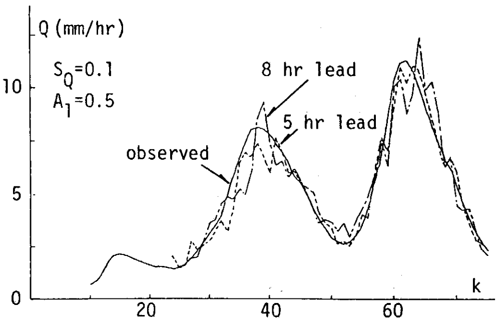
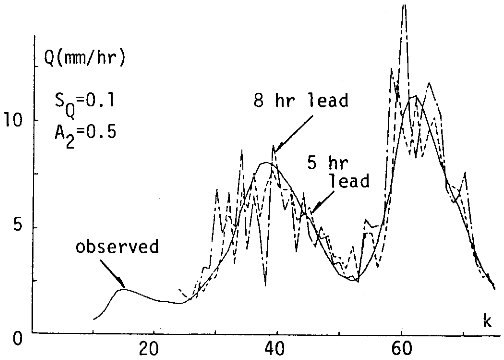


Fig. 8. Types of rainfall forecasting.

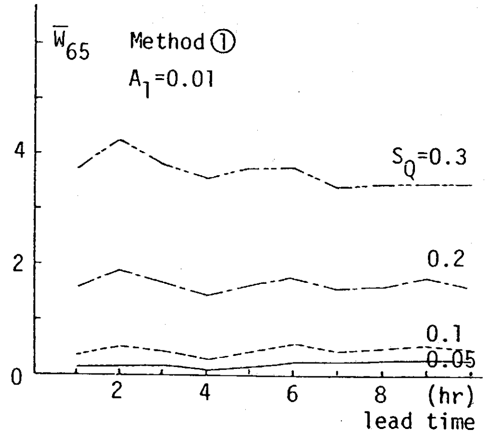


(a) Case by the method (1).

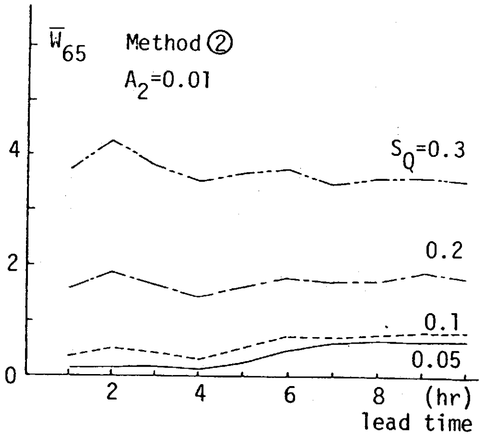


(c) Case by the method (2).

Fig. 9. Comparison between the flood prediction and the observed (simulated) discharge.

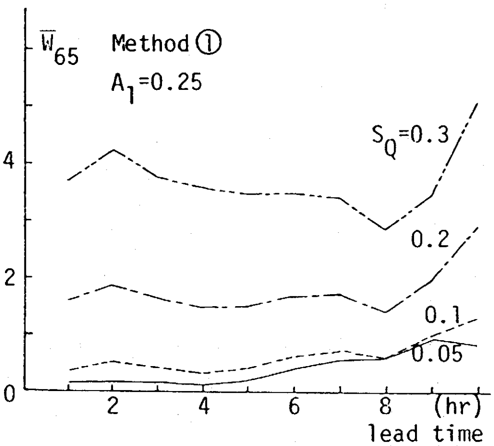


(a) Case by the method (1).

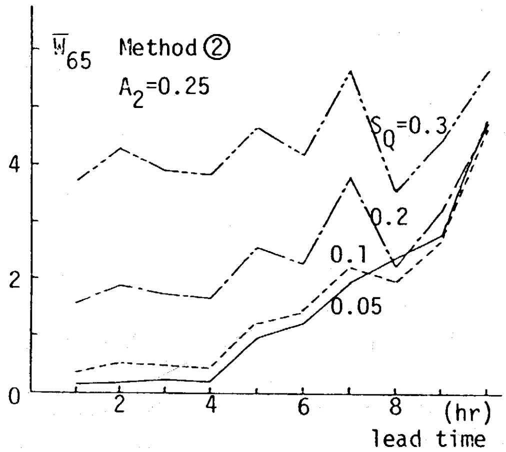


(b) Case by the method (2).

Fig. 10. Variances of prediction error, $\overline{\mathbf{W}}_{65}$.

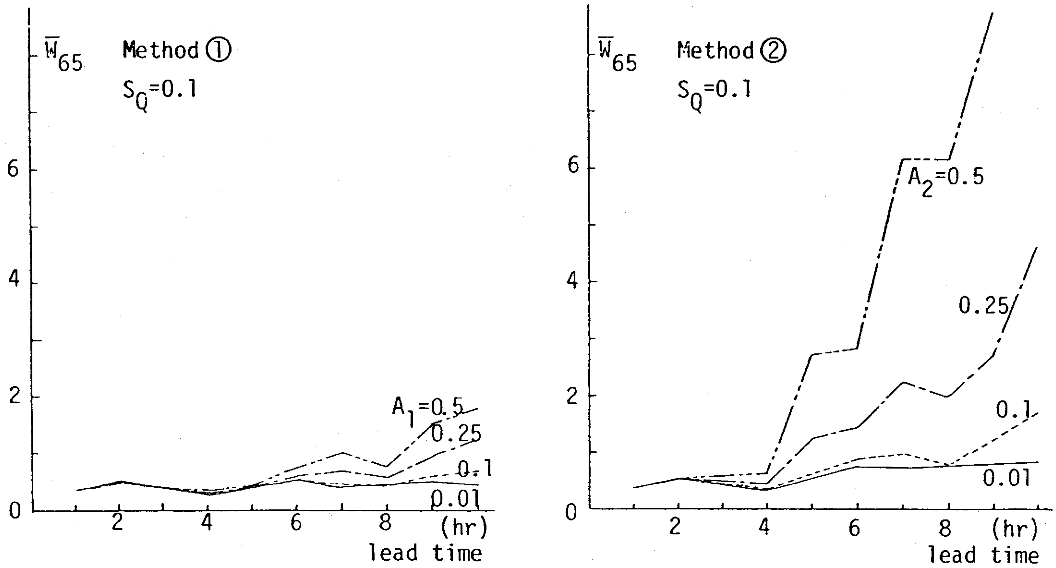


(a) Case by the method (1).



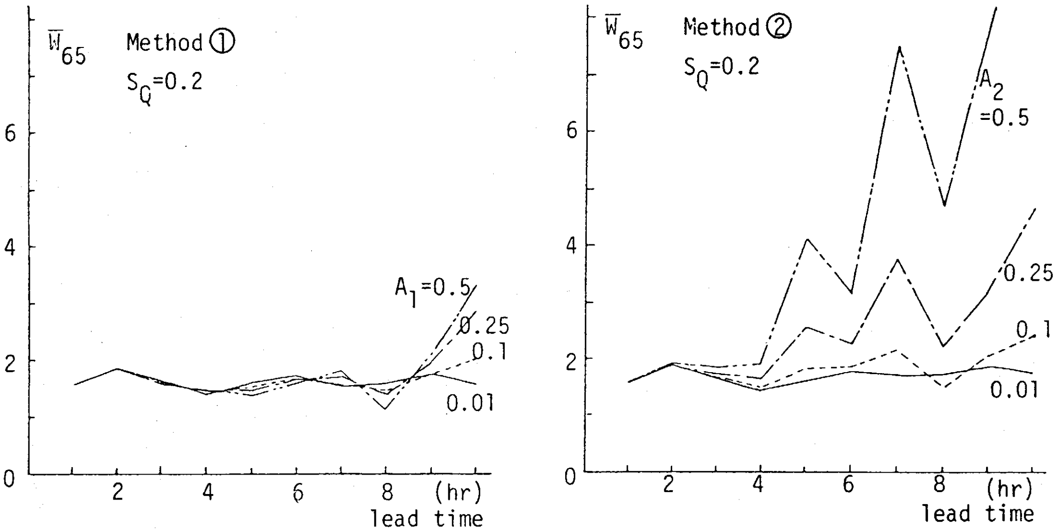
(b) Case by the method (2).

Fig. 11. Variances of prediction error, $\overline{\mathbf{W}}_{65}$.



(a) Case by the method (1). (b) Case by the method (2).

Fig. 12. Variances of prediction error, $\bar{W}_{65}$.



(a) Case by the method (1). (b) Case by the method (2).

Fig. 13. Variances of prediction error, $\bar{W}_{65}$.

ings of flood runoff phenomena. The method was applied to the experimental basin, and the influences of the accuracy of both rainfall forecasting and discharge measurement on the flood prediction were examined. The results of the application showed the method offers a promising general way of flood prediction at more complicated basins. In most river basins, more exact discussions such as the error of the hydrologic model and discharge measurement should remain. The series of results on the influences of the forecasting and measurement on the flood predictions, however, should be a good reference for further investigations on flood predictions.

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