

# DYNAMIC RESPONSE OF POROELASTIC HALF-PLANE UNDER SURFACE MOVING EXCITATIONS

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## Abstract

On the basis of Biot's dynamical theory of poroelasticity, the disturbances produced by an impulsive line load applied normal to the surface and impulsive shearing stress in a porous elastic half-space are studied. Three cases are considered; (a) Subsonic Case. (b) Supersonic Case. (c) Transonic Case. Laplace-Fourier transform is utilized for solving displacement and stress potentials in terms of which the displacements and stresses in the interior of the half-space are obtained by Cagniard's technique. In each case, the closed-form solutions for displacements and stresses are obtained. The displacements and stresses are expressed in terms of six algebraic terms; three of which are identified as the disturbances due to the specific wave fronts and the others represent the head wave contributions. Fatt's values for poroelastic parameters are utilized in the numerical calculations for Subsonic Case only. From these numerical results, comparisons are made to the disturbances produced in the classical elastic material.

## 1. INTRODUCTION

The problem of dynamical and moving loads on the surface of poroelastic material is of considerable and theoretical interest in connection with soil mechanics, petroleum prospecting and flow of fluids through sand. The wave produced by moving vehicles, impacts and other types of dynamical loads on a foundation of poroelastic materials in nature falls into this category.

Poroelastic materials are two-phase systems consisting of a porous, elastic solid phase filled with a Newtonian viscous fluid phase.

The theory of poroelastic media has its origin in the one-dimensional theory of consolidation formulated by Terzaghi<sup>1)</sup> which was later generalized by Biot<sup>2-6)</sup> in a series of his papers. Studies by Biot (1956)<sup>6)</sup> of the field equations governing the propagation of small-amplitude disturbances in an isotropic,

elastic, fluid-filled porous medium have revealed the existence of three body waves; one equivoluminal wave and two dilatational waves. The two dilatational waves are called the dilatational wave of the first kind and second kind, respectively. In the absence of dissipation, these waves are elastic in nature, the propagation being at constant velocity and with undiminished amplitude. If friction at the soil-liquid interface is taken into account, each of the wave is dispersive and dissipative, i.e the velocity is a function of the frequency and the amplitude under spatial attenuation (for a given frequency). In this paper, the Non-Dissipative Case will be considered.

A number of authors have utilized and applied the dynamical theory of Biot to some special problems. For examples, Deresiewicz<sup>7)</sup>, Deresiewicz and Rice<sup>8)</sup>, and Jones<sup>9)</sup> have applied it to study the effect of boundaries on wave propagation in fluid-filled porous soil for various cases. Paul<sup>10)</sup> has investigated the problem of the displacement produced by a line load applied normal to the bounding surface of the half-space with impulsive time depen-

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dence. Pal<sup>11)</sup> has investigated the disturbance produced by an impulsive shearing load acting on the surface which is assumed to move with a uniform velocity after creation. Paul has evaluated the displacement in the solid but not numerically. On the other hand, Pal has evaluated the surface displacement numerically. But his results seem to have some erroneous treatments in the process of the theoretical reductions.

In this paper, the dynamic response due to the following types of loadings are investigated ;

1. A line load applied normally to the bounding surface of the half-space with a constant moving velocity after creation at an arbitrary time.
2. A line load applied tangentially to the surface with a constant moving velocity after creation at an arbitrary time.

Further, three more cases are considered for each type of loading mentioned above characterized by the velocity of the moving loads.

- (a) Subsonic Case. (The load is moving more slowly than either the equivoluminal or dilatational wave speeds of the poroelastic medium).
- (b) Supersonic Case. (The load speed in greater than either wave speeds).
- (c) Transonic Case. (The load speed is between the two wave speeds).

In the analysis, the displacements and stresses are expressed in terms of four displacement potentials as in Deresiewicz and Rice<sup>8)</sup> followed by the application of the Laplace-Fourier transform to solve the displacement and stress potentials in the transformed space. The Cagniard's technique modified by Gakenheimer<sup>12)</sup> is utilized to evaluate the displacements and stresses. Expressions for displacements and stresses for three cases are obtained.

Finally, numerical formulas for Subsonic Case are explored numerically using values of material constants computed from experimental data reported by Fatt<sup>13)</sup> for a kerosine-filled sandstone. The exact closed solutions are obtained and typical examples of numerical results with the passage of time after the load starting to act are shown. The numerical results obtained are the compared to those of the classical elastic materials which were

investigated by Higuchi and Hirashima<sup>14)</sup> and discuss the differences observed.

## 2. FORMULATION OF PROBLEMS

The stress field of the porous material is denoted by

$$\begin{pmatrix} \sigma_{xx} + \sigma_p & \sigma_{xy} & \sigma_{xz} \\ & \sigma_{yy} + \sigma_p & \sigma_{yz} \\ sym. & & \sigma_{zz} + \sigma_p \end{pmatrix} \dots\dots\dots (2.1)$$

The  $\sigma_{xx}$ ,  $\sigma_{xy}$ , etc., components represent the force acting on the solid part of the faces of a unit cube of bulk material while  $\sigma_p$  represents the force applied to the fluid part.

The average displacement vector of the solid has components  $u$ ,  $v$ ,  $w$ , and that of the fluid  $U$ ,  $V$ ,  $W$ . Following the Biot's paper, we introduce the strain components for the solid as

$$e_{xx} = \frac{\partial u}{\partial x}, e_{xy} = \frac{\partial u}{\partial y} + \frac{\partial v}{\partial x}, \text{ etc. } \dots\dots\dots (2.2)$$

The only relevant strain component of the fluid is the dilatation :

$$\epsilon = \frac{\partial U}{\partial x} + \frac{\partial V}{\partial y} + \frac{\partial W}{\partial z}. \dots\dots\dots (2.3)$$

We also introduce the dilatation of the solid as

$$e = e_{xx} + e_{yy} + e_{zz} = \frac{\partial u}{\partial x} + \frac{\partial v}{\partial y} + \frac{\partial w}{\partial z} \dots\dots\dots (2.4)$$

The stress-strain relations for isotropic poroelastic material are given by,

$$\left. \begin{aligned} \sigma_{xx} &= 2Ne_{xx} + De + Q\epsilon, \\ \sigma_{yy} &= 2Ne_{yy} + De + Q\epsilon, \\ \sigma_{zz} &= 2Ne_{zz} + De + Q\epsilon, \\ \sigma_{xy} &= Ne_{xy}, \\ \sigma_{yz} &= Ne_{yz}, \\ \sigma_{xz} &= Ne_{xz}, \sigma_p = Qe + R\epsilon. \end{aligned} \right\} \dots\dots\dots (2.5)$$

Material constants  $N$ ,  $D$ ,  $Q$  and  $R$  are elastic moduli and non-negative.  $D$  and  $N$  correspond to  $\lambda$  and  $\mu$ , respectively.

The surface of the porous elastic half-space is taken as  $z = 0$ , the  $z$ -axis being drawn into the medium as shown in Fig. 1.

The equations of motion in the case of two dimensional motion in the  $xz$ -plane are given by Deresiewicz(1960)<sup>7)</sup> as

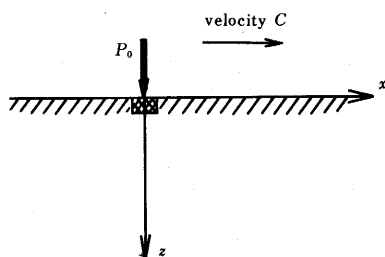


Fig. 1 Moving normal load  $P_0$  with velocity  $C$  on the surface.

$$\left. \begin{aligned} P\nabla^2\phi + Q\nabla^2\psi &= \frac{\partial^2}{\partial t^2}(\rho_{11}\phi + \rho_{12}\psi), \\ Q\nabla^2\phi + R\nabla^2\psi &= \frac{\partial^2}{\partial t^2}(\rho_{12}\phi + \rho_{22}\psi). \end{aligned} \right\} \dots\dots\dots (2.6)$$

$$\left. \begin{aligned} N\nabla^2H &= \frac{\partial^2}{\partial t^2}(\rho_{11}H + \rho_{12}G), \\ 0 &= \frac{\partial^2}{\partial t^2}(\rho_{12}H + \rho_{22}G). \end{aligned} \right\} \dots\dots\dots (2.7)$$

where  $P = D + 2N$  and  $\nabla^2$  is the Laplacian and the potentials  $\phi$ ,  $\psi$ ,  $H$  and  $G$  are related to the displacements  $u$ ,  $w$ ,  $U$  and  $W$  by the following equations.

$$\left. \begin{aligned} u &= \phi_{,x} + H_{,z}, \quad w = \psi_{,z} - H_{,x}, \\ U &= \phi_{,x} - G_{,z}, \quad W = \psi_{,z} - G_{,x} \end{aligned} \right\} \dots\dots\dots (2.8)$$

The dynamic coefficients ( $\rho_{11}$ ,  $\rho_{12}$ ,  $\rho_{22}$ ) are related to the mass per unit volume of aggregate,  $\rho_{(s)}$  and  $\rho_{(l)}$ , of the solid and liquid, respectively, by means of

$$\left. \begin{aligned} \rho_{(s)} &= \rho_{11} + \rho_{12}, \\ \rho_{(l)} &= \rho_{12} + \rho_{22}, \end{aligned} \right\} \dots\dots\dots (2.9)$$

and satisfy the inequalities

$$\left. \begin{aligned} \rho_{11} &> 0, \quad \rho_{22} > 0, \quad \rho_{12} < 0, \\ \rho_{11}\rho'_{22} - \rho_{12}^2 &> 0. \end{aligned} \right\} \dots\dots\dots (2.10)$$

Define the Laplace-Fourier transform of a function  $u(x, z, t)$  as

$$\left. \begin{aligned} u^*(k, z; t) &= \int_{-\infty}^{\infty} u(x, z, t) \cdot \exp(-ikx) dx, \\ \bar{u}^*(k, z; p) &= \int_0^{\infty} u^*(k, z, t) \cdot \exp(-pt) dt. \end{aligned} \right\} \dots\dots\dots (2.11)$$

For the case of surface excitation only and for non-zero pervious surface (i.e.  $\sigma_0(x, t) \neq 0$ ), the boundary conditions are

$$\left. \begin{aligned} \tau_{xz} &\equiv \sigma_{xz} + \sigma_p = \sigma(x; t), \\ \sigma_{xz} &= \tau(x; t), \quad \sigma_p = \sigma_0(x; t). \end{aligned} \right\} \dots\dots\dots (2.12)$$

These can be rearranged as

$$\left. \begin{aligned} \sigma_{zz} &= \sigma(x; t) - \sigma_0(x; t), \\ \sigma_{xz} &= \tau(x; t), \quad \sigma_p = \sigma_0(x; t). \end{aligned} \right\} \dots\dots\dots (2.13)$$

By applying the Laplace-Fourier transform to Eq. (2.6), Eq. (2.7), Eq. (2.8), Eq. (2.12) or Eq. (2.13), we can write the explicit forms of transformed displacements and stresses for the case of surface excitation only which can be written as follows.

$$\left. \begin{aligned} \bar{u}^* &= ik[A_2 \exp(-\xi_1 z) + A_4 \exp(-\xi_2 z)] \\ &\quad + B_2 \xi_3 \exp(-\xi_3 z), \\ \bar{w}^* &= -A_2 \xi_1 \exp(-\xi_1 z) - A_4 \exp(-\xi_2 z) \\ &\quad + ikB_2 \exp(-\xi_3 z), \\ \bar{U}^* &= ik[A_2 \beta_1 \exp(-\xi_1 z) + A_4 \beta_2 \exp(-\xi_2 z)] \\ &\quad - (\gamma_{12}/\gamma_{22}) B_2 \xi_3 \exp(-\xi_3 z), \\ \bar{W}^* &= -A_2 \beta_1 \xi_1 \exp(-\xi_1 z) - A_4 \beta_2 \xi_2 \exp(-\xi_2 z) \\ &\quad - ik(\gamma_{12}/\gamma_{22}) B_2 \exp(-\xi_3 z), \\ \bar{\sigma}_{xx}^* &= [-(D_1 + 2N)k^2 + D_1 \xi_1^2] A_2 \exp(-\xi_1 z) \\ &\quad + [(-D_2 + 2N)k^2 + D_2 \xi_2^2] A_4 \exp(-\xi_2 z) \\ &\quad + 2iNk\xi_3 B_2 \exp(-\xi_3 z), \\ \bar{\sigma}_{zz}^* &= [-D_1 k^2 + (D_1 + 2N)\xi_1^2] A_2 \exp(-\xi_1 z) \\ &\quad + [GD_2 k^2 + (D_2 + 2N)\xi_2^2] A_4 \times \\ &\quad \exp(-\xi_2 z) - 2iNk\xi_3 B_2 \exp(-\xi_3 z), \\ \bar{\sigma}_{xz}^* &= -2A_2 iNk\xi_1 \exp(-\xi_1 z) \\ &\quad - 2A_4 iNk\xi_2 \exp(-\xi_2 z) \\ &\quad - N(k_2 + \xi_3^2) B_2 \exp(-\xi_3 z), \\ \bar{\sigma}_{xy}^* &= -D_1(k^2 - \xi_1^2) A_2 \exp(-\xi_1 z) \\ &\quad - D_2(k^2 - \xi_2^2) A_4 \exp(-\xi_2 z), \\ \bar{\sigma}_p^* &= -Q_1(k^2 - \xi_1^2) A_2 \exp(-\xi_1 z) \\ &\quad - Q_2(k^2 - \xi_2^2) A_4 \exp(-\xi_2 z). \end{aligned} \right\} \dots\dots\dots (2.14)$$

where,

$A_2$ ,  $A_4$ ,  $B_2$  = constants which can be determined from boundary conditions of loadings.

$$D_1 = D + Q\beta_1, \quad D_2 = D + Q\beta_2, \quad Q_1 = Q + R\beta_1,$$

$$Q_2 = Q + R\beta_2,$$

$$\xi_1^2 = k^2 + \delta_2, \quad \delta_i^2 = \eta^2 p^2, \quad (i = 1, 2, 3)$$

$$\eta_1^2 = \frac{\rho}{2AH}(B - \sqrt{B^2 - 4AC}),$$

$$\eta_2^2 = \frac{\rho}{2AH}(B + \sqrt{B^2 - 4AC}), \quad \eta_3^2 = \frac{\rho C}{N\gamma_{22}},$$

$$\bar{B} = (2AHk^2 + Bp^2)/\rho p^2,$$

$$\bar{C} = (AH^2 k^2 + BH p^2 k^2 + \rho^2 C p^4)/\rho^2 p^4,$$

$$A = \rho_{11}\rho_{12} - \rho_{12}^2, \quad B = \gamma_{11}\rho_{22} + \gamma_{22}\rho_{11} - 2\gamma_{12}\rho_{12},$$

$$\begin{aligned}
 C &= \gamma_{11}\gamma_{22} - \gamma_{12}^2, \\
 H &= P + R + 2Q, \quad \rho_{11} = P/H, \quad \rho_{22} = R/H, \\
 \rho_{12} &= Q/H, \quad \gamma_{11} = \rho_{11}/\rho, \quad r_{22} = \rho_{22}/\rho, \\
 \gamma_{12} &= \rho_{12}/\rho, \\
 \beta_1 &= -\left[ \frac{Q\delta_1^2 - p^2\rho_{12}}{R\delta_1^2 - p^2\rho_{22}} \right], \quad \beta_2 = -\left[ \frac{Q\delta_2^2 - p^2\rho_{12}}{R\delta_2^2 - p^2\rho_{22}} \right].
 \end{aligned}$$

$$\begin{Bmatrix} -D_1k^2 + (D_1 + 2N)\xi_1^2 & -D_2k^2 + (D_2 + 2N)\xi_2^2 & -2dNk\xi_3 \\ 2iNk\xi_1 & 2iNk\xi_2 & N(k^2 + \xi_3^2) \\ Q_1(k^2 - \xi_1^2) & Q_2(k^2 - \xi_2^2) & 0 \end{Bmatrix} \begin{Bmatrix} A_2 \\ A_4 \\ B_2 \end{Bmatrix} = \begin{Bmatrix} \bar{\sigma}^* - \bar{\sigma}_0^* \\ -\bar{\tau}^* \\ -\bar{\sigma}_0^* \end{Bmatrix} \dots\dots (2.15)$$

### 3. BOUNDARY CONDITIONS AND THEIR SOLUTIONS

Since the operations to determine the solutions of displacements  $u$ ,  $w$ ,  $U$  and  $W$  and the stresses  $\sigma_{xx}$ ,  $\sigma_{zz}$ ,  $\sigma_{xz}$ ,  $\sigma_{yy}$  and  $\sigma_p$  are similar, here we mention only the solution for  $\sigma_{xx}$  as a typical example to be discussed in each case.

**CASE 1:** A line load  $P_0$  applied normal to the surface of the medium with a constant moving velocity after creation. (Fig. 1)

The boundary conditions are given by,

$$\left. \begin{aligned} \sigma_{zz}(x, 0; t) &= -P_0\delta(x - Ct) \cdot H(t), \\ \sigma_{zx}(x, 0; t) &= 0, \\ \sigma_p(x, 0; t) &= 0. \end{aligned} \right\} \dots\dots (3.1)$$

where  $H(t)$  is the Heaviside unit-step function,  $\delta(t)$  is the impulsive (Dirac) delta function and  $P_0$  is a constant.

Applying the Laplace transform to Eq. (3.1) and substituting into Eq. (2.11), we obtain the Laplace-Fourier transform of displacements and stresses. Then, operations of inverse Fourier transform are carried out and the Laplace transform of displacements and stresses were obtained.

The Fourier inverse transform of  $\sigma_{xx}$  is

$$\begin{aligned}
 \bar{\sigma}_{xx}^p &= \frac{1}{2} \cdot \int_{-\infty}^{\infty} F_x^p(\alpha) \exp[-(p/V_1) \\
 &\quad \cdot (m_\beta z - i\alpha x)] d\alpha, \quad (\beta = 1, 2, 3) \dots\dots\dots (3.2)
 \end{aligned}$$

In expression Eq. (3.2), the parameter  $p$  only appears in the argument of exponential through  $p/V_1$ . This form is necessary to use Cagniard's technique. By using this technique, we can obtain the solutions for Subsonic Case. With some modifications and alterations on the solutions for Subsonic Case, we can obtain the solutions for the Supersonic and Transonic Cases. The explicit solu-

For the boundary conditions mentioned in Eq. (2.13), we can deduce their explicit forms in the form of matrix as

tions are as follows.

**Subsonic Case.**

$$\begin{aligned}
 \sigma_{xx} &= H(t - r/V_1) \cdot Re[\dot{a}_1 \cdot F_x^1(\alpha_1)] \\
 &+ H(t - r/V_2) \cdot Re[\dot{a}_{21} \cdot F_x^2(\alpha_{21})] \\
 &+ f_\theta^{22} \cdot f_t^{22} \cdot Re[\dot{a}_{22} \cdot F_x^2(\alpha_{22})] \\
 &+ f_\theta^{23} \cdot f_t^{23} \cdot Re[\dot{a}_{23} \cdot F_x^2(\alpha_{23})] \\
 &+ H(t - r/V_3) \cdot Re[\dot{a}_{31} \cdot F_x^3(\alpha_{31})] \\
 &+ f_\theta^{32} \cdot f_t^{32} \cdot Re[\dot{a}_{32} \cdot F_x^3(\alpha_{32})] \\
 &+ f_\theta^{33} \cdot f_t^{33} \cdot Re[\dot{a}_{33} \cdot F_x^3(\alpha_{33})] \quad (\equiv \sigma_{xx}^{sub}) \quad (3.3)
 \end{aligned}$$

**Supersonic Case.**

$$\begin{aligned}
 \sigma_{xx} &= \sigma_{xx}^{sub} + \frac{P_0}{2\pi[(l_2^2 - 2l^2)(H_1 - H_2) + H_3]} \times \\
 &\times \left\{ -f_\theta^3 \cdot (D_1 + Q\beta_1 - 2Nl^2) \cdot Q_2 l_1^2 (l_2^2 - 2l^2) \right. \\
 &\quad \cdot \delta\left(x - Ct + \frac{z}{l}\sqrt{1 - l^2}\right) \\
 &\quad + f_\theta^4 \cdot (D_2 l_1^2 + 2Nl^2) \cdot Q_1 (l_2^2 - 2l^2)^2 \\
 &\quad \cdot \delta\left(x - Ct + \frac{z}{l}\sqrt{l_1^2 - l^2}\right) \\
 &\quad + f_\theta^5 \cdot (P_1 - P_2 \sqrt{l_2^2 - l^2}) \cdot 2Nl^2 \\
 &\quad \cdot \delta\left(x + Ct + \frac{z}{l}\sqrt{l_1^2 - l^2}\right) \Big\} \\
 &+ \frac{P_0}{2\pi(V_1 + Cl)[(l_2^2 - 2l^2)(H_1 - H_2) + H_3]} \cdot \\
 &\cdot \left\{ f_\theta^5 \cdot (D_2 l_1^2 + 2Nl^2) \cdot Q_1 (l_2^2 - 2l^2)^2 \right. \\
 &\quad \times \delta\left(x - Ct - \frac{z}{l}\sqrt{l_2^2 - l^2}\right) + 2f_\theta^6 \cdot Nl_2 \sqrt{l_2^2 - l^2} \cdot \\
 &\quad \cdot (P_1 - P_2) \cdot \delta\left(x + Ct - \frac{z}{l}\sqrt{l_2^2 - l^2}\right) \Big\} \quad \dots (3.4)
 \end{aligned}$$

**Transonic Case.**

$$\sigma_{xx} = \sigma_{xx}^{sub} + \frac{P_0}{2\pi[(l_2^2 - 2l^2)(H_1 - H_2) + H_3]} \times$$

$$\begin{aligned}
& \times \left\{ f_{\theta}^4 \cdot (D_2 l_1^2 + 2Nl^2) \cdot Q_1 (l_2^2 - 2l^2)^2 \right. \\
& \cdot \delta \left( x - Ct + \frac{z}{l} \sqrt{l_1^2 - l^2} \right) \\
& + f_{\theta}^5 \cdot (D_2 l_1^2 + 2Nl^2) Q_1 (l_2^2 - 2l^2)^2 \\
& \cdot \delta \left( x - Ct - \frac{z}{l} \sqrt{l_1^2 - l^2} \right) \\
& + 2f_{\theta}^6 \cdot (P_1 - P_2 \sqrt{l_2^2 - l^2}) \cdot Nl^2 \\
& \cdot \delta \left( x + Ct + \frac{z}{l} \sqrt{l_2^2 - l^2} \right) \Big\} \\
& + \frac{P_0 \cdot f_{\theta}^7 Nl^2 \sqrt{l_2^2 - l^2} \cdot (P_1 - P_2) \cdot \delta \left( x + Ct - \frac{z}{l} \sqrt{l_2^2 - l^2} \right)}{\pi (V_1 + Cl) [(l_2^2 - 2l^2)(H_1 + H_2) + H_3]} \quad (3.5)
\end{aligned}$$

**CASE 2:** A line load  $Q_0$  applied tangentially to the surface of the medium with a constant moving velocity after creation. (Fig. 2)

The boundary conditions are given by,

$$\left. \begin{aligned} \sigma_{zz}(x, 0; t) &= 0, \\ \sigma_{xz}(x, 0; t) &= -Q_0 \cdot \delta(x - ct) \cdot H(t), \\ \sigma_p(x, 0; t) &= 0. \end{aligned} \right\} \dots\dots\dots (3.6)$$

As similar treatments in **Case 1** can be carried out for this case, the results of the solutions are omitted here owing to the limited space.

In Eq. (3.3) to Eq. (3.5), the following notations are used.

$$\left. \begin{aligned} H_1 &= Q_2 l_1^2 (D_1 + 2N - 2Nl^2), \\ H_2 &= Q_1 [(D_2 + 2N)l_1^2 - 2Nl^2], \\ H_3 &= 4Nl^2 \sqrt{l_2^2 - l^2} (l_2^2 - 2l^2) \cdot (Q_2 l_1^2 \sqrt{1 - l^2} \\ &\quad - Q_1 \sqrt{l_1^2 - l^2}), \\ P_1 &= Q_2 l_1^2 (l_2^2 - l^2) \sqrt{1 - l^2}, \\ P_2 &= Q_1 (l_2^2 - 2l^2) \sqrt{l_1^2 - l^2}, \\ F_x^1(a) &= -Q_2 l_1^2 m_0^2 (D_1 - 2Na^2) \cdot P_0 / [(V_1 + iCa)R_0 N], \\ F_x^2(a) &= 2Q_1 m_0^2 \cdot D_1 Na^2 \\ &\quad \cdot P_0 / [(V_1 + iCa)R_0 N], \\ F_x^3(a) &= 4Na^2 m_3 m_0 (Q_1 m_2 - Q_2 l_1^2 m_1) \\ &\quad \cdot P_0 / [(V_1 + iCa)R_0 N], \\ R_0 &= m_0^2 m_4 / N + 4a^2 m_3 m_0 (Q_1 m_2 - Q_2 l_1^2 m_1), \\ l &= V_1 / C, \quad l_1 = V_2 / C, \quad l_2 = V_3 / C, \quad m_0 = 2a^2 + l_2^2, \\ m_1 &= \sqrt{l^2 + a^2}, \quad m_2 = \sqrt{l_1^2 + a^2}, \quad m_3 = \sqrt{l_2^2 + a^2}. \end{aligned} \right\}$$

$V_1$ , and  $V_2$  are velocities of dilatational waves of first and second kind, respectively,  $V_3$  is velocity of equivoluminal wave and  $C$  is the velocity of moving load.

$$\begin{aligned}
f_{\theta}^{22} &= \begin{cases} 1 & \text{for } 0 \leq \theta \leq \cos^{-1}(l/l_1) \\ 0 & \text{otherwise} \end{cases}, \quad f_{\theta}^{23} = \begin{cases} 1 & \text{for } \pi - \cos^{-1}(l/l_1) \leq \theta \leq \pi \\ 0 & \text{otherwise} \end{cases} \\
f_{\theta}^{32} &= \begin{cases} 1 & \text{for } 0 \leq \theta \leq \cos^{-1}(l/l_2) \\ 0 & \text{otherwise} \end{cases}, \quad f_{\theta}^{33} = \begin{cases} 1 & \text{for } \pi - \cos^{-1}(l/l_2) \leq \theta \leq \pi \\ 0 & \text{otherwise} \end{cases} \\
f_t^{22} &= \begin{cases} 1 & \text{for } (r/V_1)(\sqrt{l_1 - l} \sin \theta + \cos \theta) \leq t \leq l_1 r/V_1 \\ 0 & \text{otherwise} \end{cases} \\
f_t^{23} &= \begin{cases} 1 & \text{for } (r/V_1)(\sqrt{l_1 - l} \sin \theta - \cos \theta) \leq t \leq l_1 r/V_1 \\ 0 & \text{otherwise} \end{cases} \\
f_t^{32} &= \begin{cases} 1 & \text{for } (r/V_1)(\sqrt{l_2 - l} \sin \theta + \cos \theta) \leq t \leq l_2 r/V_1 \\ 0 & \text{otherwise} \end{cases} \\
f_{\theta}^3 &= \begin{cases} 1 & \text{for } 0 \leq \theta \leq \cos^{-1}l \\ 0 & \text{otherwise} \end{cases}, \quad f_{\theta}^4 = \begin{cases} 1 & \text{for } 0 \leq \theta \leq \cos^{-1}(l/l_1) \\ 0 & \text{otherwise} \end{cases} \\
f_{\theta}^5 &= \begin{cases} 1 & \text{for } \pi - \cos^{-1}(l/l_1) \leq \theta \leq \pi \\ 0 & \text{otherwise} \end{cases}, \quad f_{\theta}^6 = \begin{cases} 1 & \text{for } 0 \leq \theta \leq \cos^{-1}(l/l_2) \\ 0 & \text{otherwise} \end{cases} \\
f_{\theta}^7 &= \begin{cases} 1 & \text{for } \pi - \cos^{-1}(l/l_2) \leq \theta \leq \pi \\ 0 & \text{otherwise} \end{cases} \\
\alpha_1 &= \sqrt{\tau^2 - l} \sin \theta + i\tau \cos \theta, & \alpha_{21} &= \sqrt{\tau^2 - l_1^2} \sin \theta + i\tau \cos \theta \\
\alpha_{22} &= i(\tau \cos \theta - \sqrt{l_1^2 - \tau^2} \sin \theta), & \alpha_{23} &= i(\tau \cos \theta + \sqrt{l_1^2 - \tau^2} \sin \theta) \\
\alpha_{31} &= \sqrt{\tau^2 - l_2^2} \sin \theta + i\tau \cos \theta, & \alpha_{32} &= i(\tau \cos \theta - \sqrt{l_2^2 - \tau^2} \sin \theta) \\
\alpha_{33} &= i(\tau \cos \theta - \sqrt{l_2^2 - \tau^2} \sin \theta)
\end{aligned}$$

#### 4. NUMERICAL CALCULATIONS AND THEIR CONSIDERATION

Pal has carried out investigation to determine surface displacements for different values of non-dimensional quantity ( $\tau = V_1 t/r$ ), by assuming particular values of  $\gamma$  ( $\gamma = V_1/C$ ) such as 0.768, 0.70 and 0.60.

In this paper, independent parameter  $V_3 t$  is adopted instead of time variable  $t$ . Correspondingly, the dimensionless quantities corresponding to the stresses and displacements are in terms of  $V_3 t/r$ . This approach is similar to the ones carried out by Higuchi and Hirashima so that comparison with the results for classical elastic material determined by them can be done.

In this similar approach, moving point as the origin of coordinate is adopted in the calculation and analysis. For both cases, only numerical analysis for Sobsonic Cases are shown here.

In general, the wave geometry which corresponds to the dilatational and the equivoluminal contributions to the stresses and displacements can be shown graphically. To study this wave geometry, let consider  $\sigma_{xx}$  from **Case 1** with the moving velocity  $C = 0.5 V_3$  for illustration.

From Eq. (3.3), the term  $H(t-8/V_1) \cdot Re[\dot{a}_1 \cdot F_x^1(\alpha_1)]$  represents the dilatational motion of the first kind behind the cylindrical wave front at  $r = V_1 t$ ; the term  $H(t-r/V_2) \cdot Re[\dot{a}_{21} \cdot F_x^2(\alpha_{21})]$  represents the dilatational motion of the second kind behind the cylindrical wave front at  $r = V_2 t$  and the term  $H(t-r/V_3) \cdot Re[\dot{a}_{21} \cdot F_x^3(\alpha_{31})]$  represents the equivoluminal motion behind the cylindrical wave front at  $r = V_3 t$ .

Let the following notations be defined as,

$$\left. \begin{aligned} r_i &= V_i t. (i = 1, 2, 3), \quad \tau_{22} = \sqrt{l_1^2 - l^2} \sin \theta + \cos \theta, \\ \tau_{23} &= \sqrt{l_1^2 - l^2} \sin \theta - \cos \theta, \\ \tau_{32} &= \sqrt{l_2^2 - l^2} \sin \theta + \cos \theta, \\ \theta_{22} &= \cos^{-1}(l/l_1), \quad \theta_{23} = \pi - \cos^{-1}(l/l_2), \\ \theta_{32} &= \cos^{-1}(l/l_2), \quad \theta_{33} = \pi - \theta_{32}, \end{aligned} \right\}$$

The terms of  $f_\theta^{32} f_t^{22} \cdot Re[\dot{a}_{22} \cdot F_x^2(\alpha_{22})]$  and  $f_\theta^{23} f_t^{23} \cdot Re[\dot{a}_{23} \cdot F_x^2(\alpha_{23})]$  give the dilatational motions of second kind behind the plane-wave front at the surface  $\tau_{22}$  and  $\tau_{23}$  and bounded by  $\theta_{22}$  and  $\theta_{23}$ , respectively.

On the other hand, the term  $f_\theta^{32} f_t^{32} \cdot Re[\dot{a}_{32} \cdot$

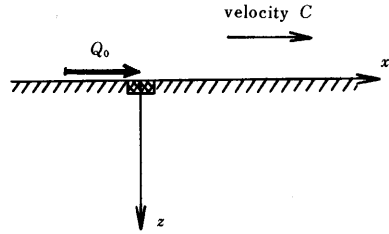


Fig. 2 Moving tangential load  $Q_0$  with velocity  $C$  on the surface.

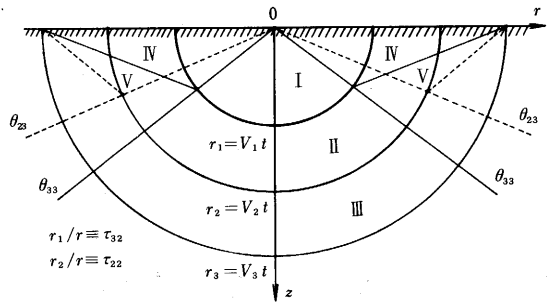


Fig. 3 Wave geometry.

$F_x^3(\alpha_{32})]$  and  $f_\theta^{33} f_t^{33} \cdot Re[\dot{a}_{33} \cdot F_x^3(\alpha_{33})]$  give the equivoluminal motions behind the plane-wave front at the surface  $\tau_{32}$  and bounded by  $\theta_{32}$  and  $\theta_{33}$ , respectively.

The above statements can be shown graphically as in **Fig. 3**. From this figure, regions I and II are the zones disturbed by the dilatational waves of the first and second kinds, respectively, and region III is the region disturbed by the equivoluminal wave. Waves in regions IV and V are called "head-waves" created by the equivoluminal and dilatational waves.

**Fig. 4** shows the contribution of each term of Eq. (3.3) for time  $T = 1$  and  $T = 10$ .

**Fig. 5** shows the distribution of stress  $\frac{\sigma_{xx}}{P_0} z$  for the case of a line load  $P_0$  which is applied normal to the boundary surface with moving velocity  $C = 0.5 V_3$  for various values of  $T$ . From this figure, we can observe that for poroelastic material, the stress seems to increase for a certain time, and as the time increases, and to leave behind, i.e. moving away from the origin of the coordinate towards the negative  $(x - Ct)/z$  axis. But in the classical elasticity case, the stress can be observed in such a way moving towards the origin of the coordinate as the time increases.

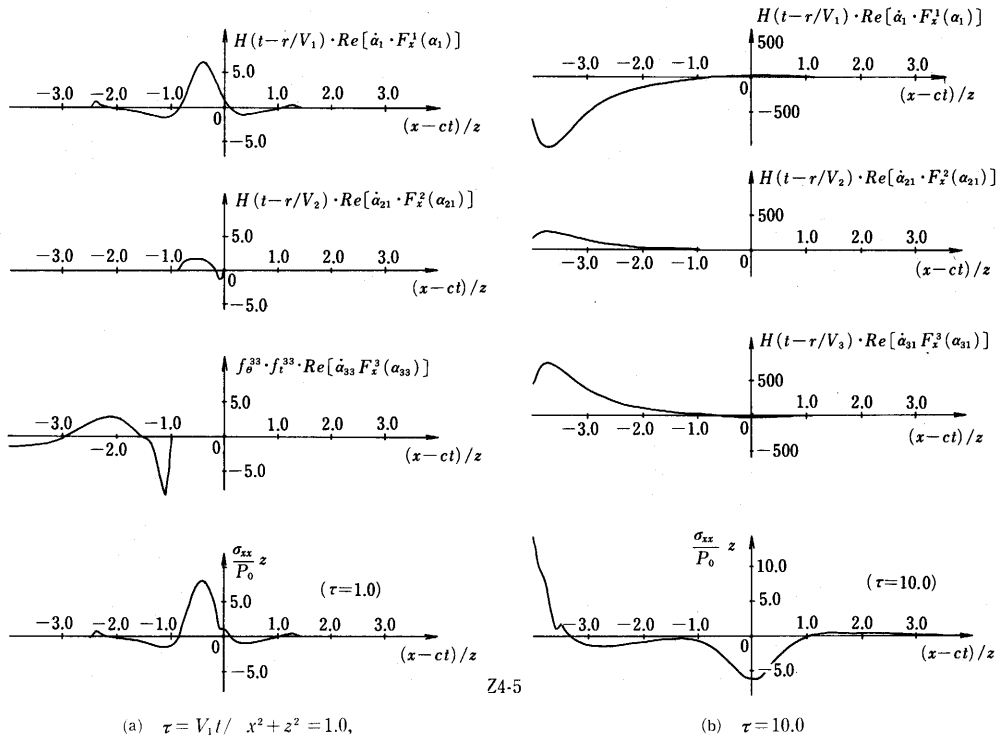


Fig. 4 Distributions of stress  $\frac{\sigma_{xx}}{P_0} z$  for the case of normal load  $P_0$  with moving velocity  $C = 0.5 V_3$ .

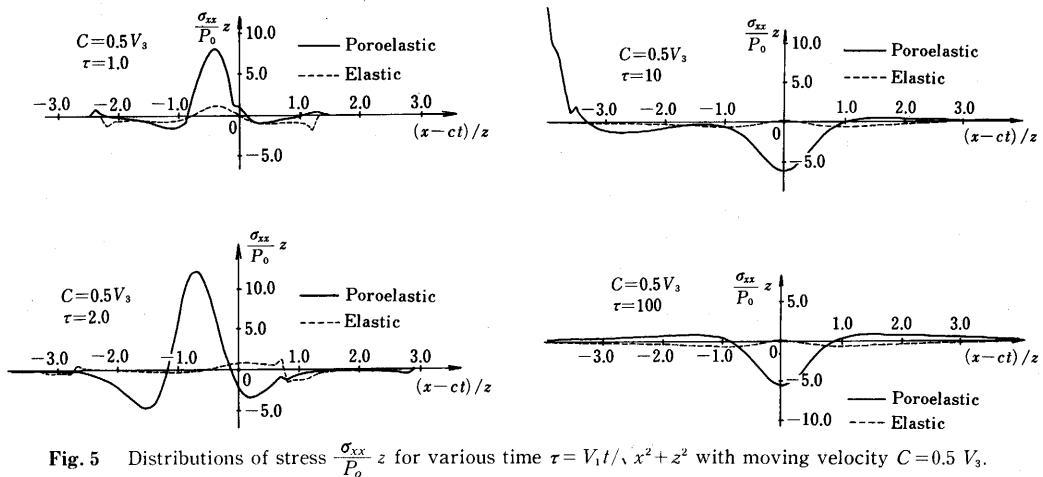


Fig. 5 Distributions of stress  $\frac{\sigma_{xx}}{P_0} z$  for various time  $\tau = V_1 t / \sqrt{x^2 + z^2}$  with moving velocity  $C = 0.5 V_3$ .

Fig. 6 shows the distribution of stress  $\frac{\sigma_{xx}}{Q_0} z$  for the case of tangential line load  $Q_0$  with moving velocity  $C = 0.9 V_3$ . The behaviour mentioned in Case 1 above also can be observed here. Another behaviour which only can be observed in Case 2 is

that the shape of the distributions of stress and displacement of poroelastic material follows the shape of stress and displacement distributions of classical elastic material as the time increases.

As mentioned by Higuchi and Hirashima that the stresses for the classical elastic material

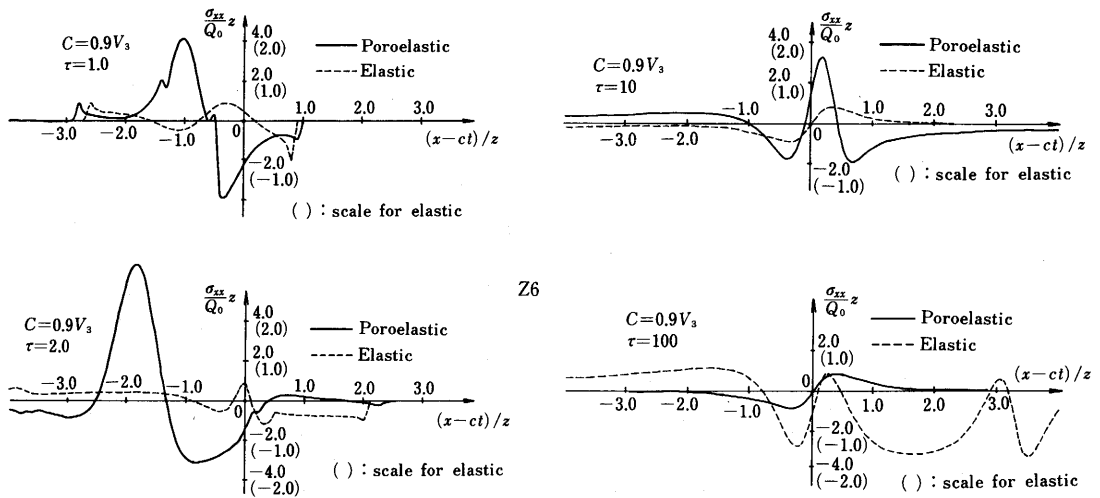


Fig. 6 Distributions of stress  $\frac{\sigma_{xx}}{Q_0} z$  for the case of tangential load  $Q_0$  with moving velocity  $C = 0.9 V_3$ .

increase as the time increases for the case of moving velocity  $C = 0.9 V_3$ ; but this behaviour does not appear in the poroelastic cases.

### 5. CONCLUDING REMARKS

We have analyzed the disturbances generated in the homogeneous, isotropic poroelastic half-space by two types of loading. We may conclude that the distributions of stress and displacement are mainly due to the disturbances by dilatational motion of first and second kinds and the equivoluminal motion behind the cylindrical waves.

We also may say that the distribution of stresses and displacements in poroelastic material are different from those in the classical elastic material even though the nature of the loadings are the same for both materials.

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