Original Report

Excitation Mechanism of Ion Bernstein Waves by Magnetosonic Wave

(Received on 30, August 1986)

M. MATSUMOTO, K. SAKAI and S. TAKEUCHI

Abstract

The present discussion gives a physical explanation to the excitation of ion Bernstein waves ($\boldsymbol{k}$, ω) by a Magnetosonic wave ($\boldsymbol{k}_0$, ω_0). This pump wave has $\boldsymbol{k}_0 \neq 0$ (non-dipole approximation) and $\omega_0 \simeq 2\Omega_0$, Ω_0 being the ion Larmor frequency in a static magnetic field $\boldsymbol{B}_0$. Here, $\boldsymbol{k}_0 \perp \boldsymbol{B}_0$ is assumed for simplicity. In this stituation, the waves propagating in the direction $\boldsymbol{B}_0 \times \boldsymbol{k}_0$ are found to be most favorably excited. The growth rates γ are calculated in the first order of magnitude k_0, and shown to be proportional to $kk_0\tilde{E}_0^2$, $\tilde{E}_0$ being the amplitude of the pump wave.

In a uniformly magnetized plasma, the ion Bernstein waves (IBW) are discussed how to be excited by a magnetosonic pump wave ($\boldsymbol{k}_0$, ω_0) which propagates perpendicularly to the static field $\boldsymbol{B}_0$. The direction of $\boldsymbol{k}_0$ is taken in the x-axis. Here we assume that $\omega_0 \simeq 2\Omega_0$, Ω_0 being the ion Larmor frequency, and the excited IBW propagate also perpendicularly to $\boldsymbol{B}_0$, for simplicity. The growth rates γ of IBW will be calculated in the first order of magnitude k_0.

The analysis is based on a method of the time-integration along the dynamical trajectories of charged particles (characteristics of the Vlesov equation) in both fields $\boldsymbol{B}_0$ and

$$\boldsymbol{E}_0(x, t) = \hat{\boldsymbol{x}}\tilde{E}_{ox} \cos \Psi_0 + \hat{\boldsymbol{y}}\tilde{E}_{oy} \sin \Psi_0,$$
$$\Psi_0 = k_0 x - \omega_0 t \quad (1)$$

This is a present form of the pump wave field. Here, $\hat{\boldsymbol{x}}$ and $\hat{\boldsymbol{y}}$ are unit vectors repectively parallel to the x- and y-axes ($\hat{\boldsymbol{y}} \mathbin{/\!/} \boldsymbol{B}_0 \times \boldsymbol{k}_0$). At this time we can obtain the following distribution $f_0(\boldsymbol{v}, t)$ of particles as a solution to the vlasov equation by using one of invariant quantities, $\mathscr{W}(t) \exp(i\Omega_0 t)$, determined from the characteristic equations, and from physical demands on the particle distribution.

$$f_0(\boldsymbol{v}, t) = \frac{n_0 m}{2\pi T_\perp} \exp\{-\frac{m}{2T_\perp}|\mathscr{W}(t)|^2\} g(v_{/\!/}) \quad (2)$$

Here, the complex variable $\mathscr{W}(t) = w_x + \mathrm{i} w_y$ correspondingly expresses the particle velocity component in the x-y plane in the oscillating frame[1]. The relation of $\boldsymbol{w} = (w_x, w_y)$ to $\boldsymbol{v}_\perp = (v_x, v_y)$, which is the particle velocity component in the laboratory frame, is dynamically derived in the first order of k_0.

The excitation of IBW ($\boldsymbol{k}$, ω) brings some perturbations to the distribution $f_0(\boldsymbol{v}, t)$. The excited IBW are considered electrostatic. Then, using the Poisson equation, the following equation for the dispertion relation of IBW is derived.[2]

$$1 = \mathrm{i} \sum_j \frac{4\pi q^2}{mk} \int d\boldsymbol{v} \int^t dt' \exp\{\mathrm{i}\omega(t-t') - \mathrm{i}\boldsymbol{k}\cdot(\boldsymbol{r}-\boldsymbol{r}')\} \frac{\boldsymbol{k}}{k}\cdot\frac{\partial}{\partial \boldsymbol{v}'} f_0(\boldsymbol{v}', t') \quad (3)$$

The t'-integration is carried out along the particle trajectory stated before. At present, we pay attention to the gradient of velocity distribution in the direction of $\boldsymbol{k}$ at the velocity of resonant particles which will make IBW unstable.

Retaining the leading term in the first order of magnitude k_0, the factors in eq. (3) are calculated as

$$\frac{\boldsymbol{k}}{k}\cdot\frac{\partial}{\partial \boldsymbol{v}'} f_0(t') = \frac{\boldsymbol{k}}{k}\cdot\frac{\partial}{\partial \boldsymbol{v}'}\boldsymbol{w}'\cdot\frac{\partial}{\partial \boldsymbol{w}'} f_0(t')$$
$$= -\frac{n_0 m}{T_\perp} w\{\cos(\varphi - \theta + \Omega_0 \tau)$$
$$+ \frac{1}{2}\frac{k_0 v_D}{\omega_0 - 2\Omega_0} \sin(\Psi_0' - \varphi - \theta$$

$$-\Omega_0\tau)\}$$

$$\times\frac{m}{2\pi T_\perp}\exp\{-\frac{m}{2T_\perp}w^2\}g(v_{/\!/}). \qquad (4)$$

Here, φ and θ are respectively the angles of $\boldsymbol{w}$ and $\boldsymbol{k}$ to the x-axis ($/\!/ \boldsymbol{k}_0$), $\tau = t-t'$, $v_D = cE_0/B_0$ and

$$\Psi_0' = \Psi_0+\omega_0\tau+\frac{k_0 w}{\Omega_0}\{\sin\varphi-\sin(\varphi+\Omega_0\tau)\}. \qquad (5)$$

which gives the relation between the phases of pump wave ($\Psi_0 = k_0x-\omega_0 t$) at t and t'. The last term on the r.h.s. of eq. (5) results from the time variation of the y-component of $\boldsymbol{w}$ which produces the particle displacement ($x-x'$). Similarly, the exponential indices in eq. (3) are calculated as

$$-\mathrm{i}\boldsymbol{k}\cdot(\boldsymbol{r}-\boldsymbol{r}') = \mathrm{i}\eta\{\cos(\Psi_0-\theta)-\cos(\Psi_0'-\theta)\} +\mathrm{i}\frac{kw}{\Omega_0}\Big[\sin(\varphi-\theta)-\sin(\varphi-\theta +\Omega_0\tau)-\frac{1}{2}\frac{k_0 v_D}{\omega_0-2\Omega_0}\{\cos(\Psi_0 -\varphi-\theta)-\cos(\Psi_0'-\varphi-\theta -\Omega_0\tau)\}\Big]. \qquad (6)$$

where $\eta = kv_D/\Omega_0$. We note here in the k_0-order terms that Ψ_0' in the η-term on the r.h.s. is given by eq. (5), while it in the other one sufficiently given by $\Psi_0+\omega_0\tau$.

The results (2)~(6) were given from Ref. 2 in some revise.

We examine the perturbations to $(\boldsymbol{k}/k)\cdot\partial f_0/\partial \boldsymbol{v}$ at the velocity of resonant particles with IBW ($\boldsymbol{k}$, ω) by the pump wave. The exponential sine and cosine are expanded by the Bessel functions for carrying out the integrations. The velocity terms depending on τ explicitly in eq. (6) are sinusoidally just in the angle difference $\pi/2$ to those in eq. (4). Therefore, the gradient (4) in the $\boldsymbol{k}$-direction is not affected, and averaged out entirely. This is basically due to the velocity components perpendicular to $\boldsymbol{k}$ which produce the components of particle displacement ($\boldsymbol{r}-\boldsymbol{r}'$) parallel to $\boldsymbol{k}$. On the other hand, the velocity perturgations implicitly contained in Ψ_0' of eq. (6) are always in the y-direction (see eq. (5)). Those perturbations nonlearly lead to the non-vanishing gradient in eq. (4), which is largest at $\theta = \pi/2$. Thus, IBW may be most favorably excited in the direction perpendicular to both $\boldsymbol{k}_0$ and $\boldsymbol{B}_0$, i.e., in the y-direction. This excitation mechanism is not substantially different from the so-called inverse Landau damping, and the growth rates $\gamma \propto kk_0\tilde{E}_0^2/\varepsilon'$ at the lowest order, ε' being the derivative of ε (dielectric constant) with respect to ω. From the complete expressions of γ, the frequencies are given as $\omega \simeq \omega_0+n\Omega_0$, ($n = 0, \pm1, \ldots$). The magnitudes of $\boldsymbol{k}$ are determined by the real part of dispersion relation at those frequencies.

References

1) P.K. Kaw: *Advances in Plasma Physics*, ed. A. Siomon and W.B. Thompson (Interscience, NY, 1976) vol. 6
2) M. Matsumoto, K. Sakai and S. Takeuchi: J. Phys. Soc. Jpn. **55**, (1986) 3093